THE SCHLIEREN-DIFFUSION METHOD OF DETERMINING THE TURBULENCE CHARACTERISTICS OF A CARRIER MEDIUM IN RAREFIED GAS SUSPENSION STREAMS

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A method is developed by which the microcharacteristics of turbulent flow can be analyzed in the case of a medium which carries streams of gaseous suspensions. The Lagrangian scales, the pulsation rate, and the rate of turbulent diffusion are determined from the heat distribution in the wake of a linear source.

An analysis of method by which measurements can be made in turbulent dispersion streams has revealed that contact methods usually applied to continuous media (thermoanemometer, thermal and dynamic turbulometers, etc.) are basically unsuitable here. This is explained by the practical unfeasibility of determining the effect of solid particles on the sensing element of the measuring device. For dispersion streams it is possible to use only those methods which are based on studying the behavior of indicators. The use here of discrete admixtures [1, 2], however, can change the flow pattern and thus considerably affect the measured quantities.

Most promising, as to the possibility of using passive admixtures, are methods based on the phenomena of turbulent diffusion from the source. These methods had not found a wide use yet for homogeneous streams, because they were based only on simplified premises in the theory of turbulent diffusion and on the measurement of really nonexistent jet "boundaries" whose location was determined only by the sensitivity of the recording instruments [3-5]. The effect of the source and of the admixture indicator on the turbulence of the stream was obvious here. In the case of pure air, it resulted in a higher pulsation rate than that recorded by a thermoanemometer [6]. Further developments in the theory of diffusion from a source into a turbulent stream [7, 8] have made it possible to determine the characteristic quantities from measurements performed at a distance from the admixture inlet and, therefore, to avoid the effect of the source on the measured characteristics. In order to completely avoid the danger of artificial turbulization, it is necessary to replace the contact transducer of admixture concentration (thermocouple, separator) by some highly sensitive contactless method. In order to develop a method of analyzing the characteristics of a carrier turbulence, it is obviously necessary to have means of detecting the effect of solid particles on the results of measurements. A thermal wake has been chosen as the passive source of admixture, because of the simplicity with which such a source of the smallest possible size could be produced experimentally and because it is so convenient to use. In this case the turbulence characteristics of the carrier medium are determined from the temperature distribution in the wake behind the source.

An analysis of various methods of contactless measurements has shown that optical methods have the greatest spatial resolving power. The stipulation that the admixture be passive, so as to ensure a small temperature drop, makes it difficult to use interferometer methods. The high degree of localization of inhomogeneities, on the other hand, causes considerable temperature gradients. This is then a reason why the Tepler schlieren method should be used. In order to determine the temperature field by the schlieren method ($\theta = f(n)$), one measures the displacement ξ in the plane of the Foucault knife, which is proportional to the gradient of the refractive index n:

$$n(x, y) = n_0\left[1 + \frac{1}{l_s f}\int_0^y \xi(x, y)\,dy\right]. \quad (1)$$

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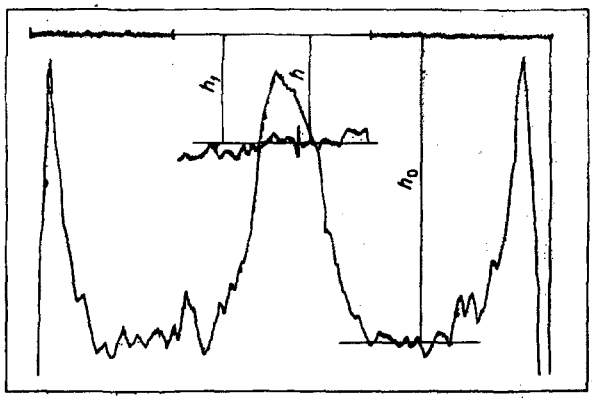

Fig. 1. Microphotogram of the cross section of a thermal wake behind a source.

Here f denotes the focal length of the instrument and n_0 is the refractive index at a segment without inhomogeneities, with the y-axis perpendicular to the illuminating aperture and to the knife edge. It follows from (1) that the longer the path of light rays in an inhomogeneity, the more they refract. In order to increase the sensitivity of an instrument to small gradients, therefore, one uses not a point source but a line source of admixture oriented parallel to the light path. The conventional method of plotting isograds from the displacement of the Foucault knife did not ensure an adequate accuracy and, therefore, the better method of absolute photometering was used. The calculation formulas in this method applied to homogeneous streams, as well as questions of sensitivity and measurement range, were discussed thoroughly in [9].

In order to perform measurements in dispersion streams, i.e., in order to determine $\xi(x, y)$ for relation (1), it is necessary to consider the effect of particles on the shadow pattern. Diffusive scattering of light at the particles results in the appearance of an additional background illumination, which for actual particles is a function of concentration only. In order to determine the effective readout point (E_0, ξ_0), it is necessary during the motion of the dispersion stream to perform pairwise luminance measurements: with the thermal wake included $(E + E_0, \xi + \xi_0)$ and excluded, respectively, matched with the displacement of the Foucault knife by $\xi_1(E_0 + E_1, \xi_0 + \xi_1)$.

Since the image is picked up within the range of normal exposures, hence the luminance can be expressed in terms of the blackening of the negative:

$$\frac{E_0}{E_0+E_1}=\frac{\xi_0}{\xi_0+\xi_1}=10^{\frac{D_0-D_1}{\gamma}}, \tag{2}$$

where γ is the film contrast factor. The blackening of the schlieren negative is determined when the latter is photometered:

$$D_0-D_1=\lg(h_1/h_0). \tag{3}$$

Here h_1 and h_0 are the distances of the photometering curves from the dark-current line. Then, according to (2) and (3),

$$\frac{\xi_0}{\xi_0+\xi_1}=\left(\frac{h_1}{h_0}\right)^{1/\gamma}, \tag{4}$$

$$\xi_0=\xi_1\frac{h_0^{-1/\gamma}}{h_1^{-1/\gamma}-h_0^{-1/\gamma}}. \tag{5}$$

Relation (5) yields the sought effective initial readout point. Analogously,

$$\frac{\xi}{\xi_0+\xi_1}=\frac{E_0+E}{E_0+E_1}-\frac{E_0}{E_0+E_1}=\left(\frac{h_1}{h}\right)^{1/\gamma}-\left(\frac{h_1}{h_0}\right)^{1/\gamma}. \tag{6}$$

Substituting for $(\xi_1 + \xi_0)$ from (5), we have

$$\xi=\xi_1\frac{h^{-1/\gamma}-h_0^{-1/\gamma}}{h_1^{-1/\gamma}-h_0^{-1/\gamma}}. \tag{7}$$

This formula can be used for determining the refraction angles of light in an inhomogeneity, on the basis of microphotograms taken from schlieren negatives of the dispersion stream. A typical such microphotogram is shown in Fig. 1. The middle hump corresponds to a thermal wake, the outer peaks correspond to diffraction maxima at the channel walls. The horizontal line in the center portion of this microphotogram was obtained during photometering of the reference negative (the aperture was opened additionally by ξ_1), and the one in the upper portion corresponds to the thermal wake. With the aid of relations (1) and (7), the field of the refractive index n was determined from the microphotograms. The relation between the temperature excess and the refractive index in an isobaric process can be expressed, according to [9], in the following form:

$$\theta(x,\ y) = \frac{n_0 - n(x,\ y)}{n(x,\ y) - 1}(273 + \theta_0). \tag{8}$$

With this formula it is possible to calculate the temperature distribution from the cross section of a dispersion stream at various distances from the heat source.

We will now briefly consider the method of determining the intensity and the turbulence scales from the turbulent diffusion characteristics. We use here the dependence of the dispersion on the distance from the source: $\langle Y^2 \rangle = f(x)$. This dependence can be found from the ratio of the moments of temperature over the cross section at various distances from the source:

$$\langle Y^2(x) \rangle = \frac{\int_0^\infty y^2 \theta(x,\ y)\, dy}{\int_0^\infty \theta(x,y)\, dy}. \tag{9}$$

The diffusion of passive admixture particles in a dispersion stream is determined as the sum total of several mechanisms acting simultaneously:

$$\langle Y^2 \rangle = \langle Y_1^2 \rangle + \langle Y_2^2 \rangle - \langle Y_3^2 \rangle + \langle Y_4^2 \rangle + \langle Y_5^2 \rangle + \langle Y_6^2 \rangle. \tag{10}$$

Here Y_1^2 represents the turbulent dispersion of carrier particles, which is of interest to us, Y_2^2 represents the effect of molecular diffusion, Y_3^2 represents the mutual effect of molecular and molar diffusion, Y_4^2 accounts for the finite dimensions of the source of the thermal wake, Y_5^2 represents the scattering of liquid particles after collision with particles of the solid component, and Y_6^2 is determined by the transport of admixture during the diffusion of solid particles. The Y_3^2 term drops out, since molecular diffusion impedes the progress of turbulent diffusion relative to the location of the admixture source. The explanation for this is that molecular diffusion tends to average out the transport velocities over a certain volume, which in turn results in a velocity lower than its instantaneous value at a point [10]:

$$\langle Y^2(\tau) \rangle = \langle Y_1^2(\tau) \rangle + 2a\tau - \frac{1}{3} a (\nabla v)^2 \tau^3 + \cdots \tag{11}$$

Let us estimate the amount which each term in expression (10) contributes to the dispersion of an admixture. According to [10], the molecular diffusion at large values of τ is

$$\langle Y_2^2 \rangle - \langle Y_3^2 \rangle = 2a\tau - A \langle Y_1^2(\tau) \rangle, \tag{12}$$

where α is the molecular thermal diffusivity and the constant $A \sim 1/RePr = Pe^{-1}$. Both these terms are much smaller than $\langle Y_1^2 \rangle$ for fully developed streams.

Thus, for example, in our experiments

$$\frac{\langle Y_2^2 \rangle}{\langle Y_1^2 \rangle} = \frac{a}{a^*} < 4\%. \tag{13}$$

Besides, the effect of $\langle Y_2^2 \rangle$ is to some measure compensated by the magnitude of $\langle Y_3^2 \rangle$. Therefore, the error incurred by disregarding molecular effects is entirely permissible. The finite size of the source will be accounted for, according to [11], by the relation

$$\langle Y_4^2 \rangle = \frac{d_0^2}{16}, \tag{14}$$

where d_0 is the source diameter.

In our experiment the diameter of the heater filament was $d_0 = 0.1$ mm and the measured value of $\langle Y^2 \rangle$ was by more than three orders of magnitude greater than the value of $\langle Y_4^2 \rangle$. When particles are present in the stream, the admixture disperses more due to a displacement of a particle by some fraction of its radius during its collision with a mole of liquid:

$$y^2 = \chi^2 \frac{d_s^2}{4}. \tag{15}$$

We hypothesize, according to [1], that the number of collisions between an element of liquid and solid particles is proportional to the ratio of the volume flow rates of the stream components. The total displacement of a liquid as a result of all "collisions" with solid particles will be

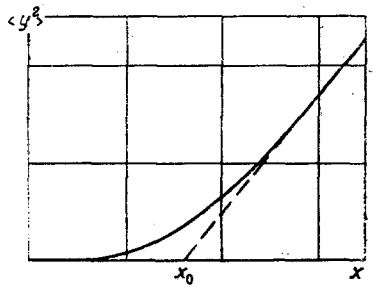

Fig. 2. Typical shape of a dispersion curve.

$$\langle Y_5^2 \rangle = \frac{3}{8} p\chi^2 v_{s1} \frac{d_s}{2} \cdot \frac{\beta}{1-\beta} \tau, \tag{16}$$

where p is the probability of a collision. For the quartz sand particles used in our experiment, $d_s = 0.6$ mm and $v_{s1} = 3.5$ m/sec. Conservatively assuming $p = 1$ and $\chi = 1/2$, we obtain $\langle Y_5^2 \rangle / \tau = 10^{-2}$ cm^2/sec for $\beta = 1\%$, which amounts to only tenths of a percent of the total value $\langle Y^2 \rangle / \tau$. It is also necessary to estimate the dispersion of an admixture due to the turbulent motion of solid particles in the stream. A comparison between $\langle Y_6^2 \rangle$ and $\langle Y_1^2 \rangle$, which has been made in [12] on the basis of spectral calculations, shows that for the kind of particles and for distances from the source measured in our tests (1-30 mm) the magnitude of $\langle Y_6^2 \rangle$ is negligibly small. It is also important to estimate the fraction of the thermal energy expended on heating the particles up within the segment of the test wake under study. Considering that this segment is triangular in cross section, we obtain for the quantity of heat accumulated by the stream components

$$Q = \frac{V_\nabla}{\tau} \rho c \delta t; \quad Q_s = \frac{V_\nabla \beta}{\tau_s} \rho_s c_s \delta t_s. \tag{17}$$

The stipulation that the hot admixture be passive is equivalent to stipulating small temperature drops and allows us to write ($t_0 = t_{s0}$):

$$\Delta t \simeq \frac{1}{2} (\delta t - \delta t_s) = \frac{Q_s}{\alpha_s F_{s\nabla}}. \tag{18}$$

The heat-transfer coefficient for the particles α_s is determined considering that the heating of particles is a transient process, and the homochronism numbers Ho are calculated from the average sojourn of a particle in a given volume: $\tau_s = l/2v_{s1}$ Equating Q_s from (17) and (18), we obtain

$$\frac{\delta t}{\delta t_s} = 1 + \frac{2V_\nabla \beta \rho_s c_s}{\alpha_s F_{s\nabla} \tau_s} = 1 + \frac{2\rho_s c_s v_s}{3l\alpha_s}. \tag{19}$$

From expressions (17) and (19) one ean easily find the fraction of the thermal energy expended on heating the particles:

$$\frac{Q_s}{Q} = \frac{\beta \rho_s c_s \tau}{\rho c \tau_s} \cdot \frac{\delta t_s}{\delta t} = \frac{c_s}{c} \mu \left[1 + \frac{2\rho_s c_s v_s}{3l\alpha_s} \right]^{-1}. \tag{20}$$

The results of calculations performed according to formula (20) for those test conditions ($Re_w = v_{s1} d_s / \nu = 160$, $v = 10$-25 m/sec, $l = 0.03$ m) show that in the studied range of discharge concentrations ($\mu < 10$ kg · sec/kg · sec) the particles accumulate less than 2% of the heat.

In this way, a comparative analysis of the terms in expression (10) shows that it is sufficiently accurate to let $\langle Y^2 \rangle = \langle Y_1^2 \rangle$. Consequently, from the experimentally determinable values of mean-squared displacement of the admixture one can find the molal dispersion of the carrier medium. The averaging in (9) may be considered here done over an infinitely long time period, since the observation time $\tau^* = 0.5$ sec chosen on the basis of photographic exposure criteria is more than three orders of magnitude longer than the largest experimentally attainable Lagrangian time scale.

The $\langle Y^2(x) \rangle$ function is closely approximated by a second-order parabola along the initial segment and it becomes a straight line at a sufficient distance (Fig. 2). Along the initial segment (small τ)

$$Ka^2 = \frac{\langle v'^2 \rangle}{v^2} = \frac{\langle Y^2(x) \rangle}{x^2}. \tag{21}$$

This simple method of determining the intensity of turbulence is not usable, however, on account of the large error due to the admixture source. At large values of τ an expression for the dispersion may be derived only for a specific model of the form $R_L(\tau)$. Approximate $\langle Y^2(\tau) \rangle$ functions, within a 1% error, obtained for every range of τ / T_L are given in [13] for the most often used forms of the correlation function. It follows from these relations that along the segment $1.5 > \tau/T_L < 100$, which is of most interest to us, the equation of the asymptote

$$\langle Y^2(\tau) \rangle = 2 \langle v'^2 \rangle T_L \left(\tau - \frac{1}{k} T_L \right). \tag{22}$$

becomes the equation of dispersion. The value of k corresponds to whatever form of $R_L(\tau)$ has been chosen. From the value of x_0 corresponding to the intersection of this asymptote with the axis of abscissas we will determine the magnitude of the Lagrangian time scale $T_L = kx_0/v$. From the slope of the extreme tangent we determine the turbulence coefficient of thermal diffusion, the Karman number, and the Lagrangian length scale:

$$a^* = \frac{v}{2}\left(\frac{d\langle Y^2\rangle}{dx}\right)_{x\to\infty} = \langle v'^2\rangle T_L, \tag{23}$$

$$\mathrm{Ka}^2 = \frac{1}{2kx_0}\left(\frac{d\langle Y^2\rangle}{dx}\right)_{x\to\infty}, \tag{24}$$

$$\Lambda = vT_L\,\mathrm{Ka} = kx_0\,\mathrm{Ka}. \tag{25}$$

From formulas (23)-(25), using the segment of the tested $\langle Y^2(x)\rangle$ curve far removed from the source, one can determine the basic macroparameters of turbulence. The necessary information about the form of the correlation function which will determine the value of k is obtained by comparing the test curve with the results of calculations based on the approximating relations in [13]. For every form of R_L one considers the corresponding values of T_L and $\langle v'^2\rangle$.

A check of the proposed method on the flow of a gas stream without particles has shown a close ($\sigma \simeq 12\%$) agreement with the data obtained by Conte–Bello [14], Laufer, and Goldenberg [5]. Tests with a quartz sand stream (D = 20 mm) suspended in air have revealed, within the 0-8 kg·sec/kg·sec range, the effect of particles on the turbulence characteristics of the carrier medium and have made it possible to obtain criterial relations for determining these quantities [12].

We emphasize, in conclusion, that the possible applications for the method described here extend beyond suspensions of solid nonreacting particles in air. At high concentrations, however, the stream ceases to be translucent and this becomes a major drawback of this method.

NOTATION

$\langle Y^2\rangle$	is the mean-square displacement;
d_s, D	are the diameters of particle and of channel, respectively;
v, v_{sl}	are the absolute and the slip velocity, respectively;
β, μ	are the actual volume and flow rate weight concentration, respectively;
c	is the specific heat;
ρ	is the density;
τ	is the time;
t	is the temperature;
a, a^*	are the molecular and molal thermal diffusivity, respectively;
Λ, T_L	are the Lagrangian turbulence scales;
R	is the correlation factor;
n	is the refractive index;
E	is the illumination intensity;
D	is the density of the negative.

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